Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Whence, to trisect an angle, construct an isosceles triangle having an arm of the angle for its side and the angle for its vertex. Draw a circle on the side of the isosceles triangle as diameter. Draw from the center of the circle a line parallel to the base of the triangle. Mark off on a straight edge a distance equal to the radius of the circle. Swing and slide the straight edge about until one end of the marked portion of the straight edge coincides with a point of the circle and the other with a point on the line while the straight edge passes through the vertex of the angle.

Also solved by S. A. JOFFE.

469A. Proposed by W. F. FLEMING, Chicago, Ill.

A pole whose length is l stands vertically against a vertical wall. A spider is at each end of the pole. The pole is drawn out from the wall in such a way that its upper end moves down the wall at a uniform rate. At the same time that the pole begins to move, the spiders begin to travel toward each other at rates equal to the rates at which the respective ends move. Determine the equations of the paths of the two spiders, in space.

SOLUTION BY H. S. UHLER, Yale University.

Let (x, y) and (x', y') denote, at any instant, the positions of the spiders which have started from the upper $(0, l)$ and lower $(0, 0)$ ends of the pole respectively. Since the spiders begin to recede from the ends of the pole at the same time that the pole starts to move, and as they crawl along the pole at the same rates as the associated ends of the pole slide along the coördinate axes, the distances which the spiders will have progressed along the pole will, at each instant, be equal to the distances which the corresponding ends of the pole have moved from their initial locations on the axes, quite regardless of whether the upper end of the pole moves uniformly or otherwise. In symbols

$$l - b = + [x^2 + (y - b)^2]^{1/2}, \tag{1}$$

$$a = + [(x' - a)^2 + y'^2]^{1/2}, \tag{1'}$$

where a and b denote the intercepts of the pole-line on the axes of x and y respectively.

Since the spiders are on the pole-line

$$\frac{x}{a} + \frac{y}{b} = 1 \quad (2), \qquad \text{and} \qquad \frac{x'}{a} + \frac{y'}{b} = 1 \quad (2').$$

Since the length of the pole is l, $a^2 + b^2 = l^2$ (3).

The analysis is now reduced to the elimination of a and b from the two sets of equations (1), (2), (3) and (1′), (2′), (3).

Equation (1), when solved for b, gives

$$b = \frac{x^2 + y^2 - l^2}{2(y - l)}. \tag{4}$$

Substituting b from (4) in (2) we find

$$a = \frac{x(x^2 + y^2 - l^2)}{x^2 - (y - l)^2}. \tag{5}$$

Substitution of a and b from (5) and (4) in (3) leads to

$$(x^2 + y^2 - l^2)^2 = 4ly(y - l)^2$$

as the equation of the path of the spider which started at the upper end of the pole.

In like manner, we find

$$a = \frac{x'^2 + y'^2}{2x'}, \qquad b = \frac{y'(x'^2 + y'^2)}{y'^2 - x'^2},$$

and

$$(x'^2 + y'^2)^2 = 2lx'(y'^2 - x'^2),$$

which is the equation of the path of the second spider.

The spiders will pass at the point

$$x = \frac{l}{2}(\sqrt{2} - 1), \qquad y = \frac{l}{2},$$

which divides the pole into upper and lower segments equal to $(l/2)(2-\sqrt{2})$ and $(l/2)\sqrt{2}$ respectively, the pole then forming an isosceles triangle with the axes.

Excellent solutions were received from Frank Irwin, J. A. Caparo, H. C. Feemster, and N. P. Pandya.

CALCULUS.

384. Proposed by JOSEPH B. REYNOLDS, Lehigh University.

In what time will a sum of money double itself at 6 per cent. interest per annum if compounded at indefinitely short intervals?

Solution by H. L. Agard, Williams College.

If the interest is compounded k times a year, the amount after n years is given by the formula

$$A = P\left(1+\frac{r}{k}\right)^{nk}.$$

When the interest is compounded at indefinitely short intervals,

$$A = \lim_{k=\infty} P\left(1+\frac{r}{k}\right)^{nk} = \lim_{k=\infty}\left[P\left(1+\frac{r}{k}\right)^{k/r}\right]^{nr} = Pe^{nr}. \tag{1}$$

In (1), setting $A = 2P$, $r = .06$ and solving for n, we have

$$n = \frac{\log_e 2}{.06} = \frac{.69315}{.06} = 11.5525 \text{ years.}$$

Also solved by H. C. Feemster, W. W. Burton, G. W. Hartwell, C. E. Flanagan, J. W. Clawson, Frank R. Morris, H. S. Uhler, Elizabeth Ovin, F. Fordero (Seville, Spain), and and Herbert N. Carleton.

386. Proposed by HERBERT N. CARLETON, West Newbury, Mass.

C is a fixed point on the perpendicular bisector of a line segment AB. Locate a point D also on this bisector, such that $AD + BD + DC$ shall be a minimum.

Solution by H. C. Feemster, York College, Nebraska.

Let the foot of the perpendicular be E, $CE = b$, $AE = EB = a$, and $DE = x$. Then $AD + BD + DC = 2\sqrt{a^2+x^2} + b - x$, which is to be a minimum. Taking the derivative of this expression, setting it equal to 0, and solving for x, we have

$$x = \frac{a\sqrt{3}}{3} = DE, \text{ as required.}$$

MECHANICS.

301. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

A wire is hanging from two points in the same horizontal plane. If the difference between the length of the wire and the actual distance between the supports is very small, show that

$$s = x\left(1+\frac{x^2}{6c^2}\right),$$

where s is one half of the length of the wire, c is the tension at the lowest point divided by w the load per unit of horizontal distance, and x is the distance of the lowest point of the curve to the point of support.